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Two-parameter techniques of numerical forecasting applied to a small intense depression

by

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(Meteorological Office)

AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE

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1. Introduction

From time to time small depressions deepen very rapidly over the Atlantic and when such a development takes place near to the coasts of the British Isles the forecaster is usually unable to foresee the coming changes until large barometric tendencies appear. Important errors in the forecasting of the gales and associated bad weather occur. It is relevant to enquire whether the newly developed techniques of numerical forecasting can be expected materially to assist the forecaster on these occasions.

2. The depression of 28-29 July 1956

A suitable case for study occurred on 28-29 July 1956. The synoptic situation at 1500 GMT 28 July 1957 is illustrated in Fig. 1. At that time a rather shallow depression was centred about 250 miles west-south-west of Cornwall with a trough of low pressure extending northeastward over most of the British Isles. Over the previous twelve hours the depression had moved northeastward at about 25 kt and had deepened slowly, the central pressure falling from 1000 mb to about 994 mb. Although at 1200 GMT there were a number of ships around the centre none reported winds in excess of 20 kt and it was very difficult to forecast the coming gales with confidence. The depression lay just to the east of a trough in the 1000-500 mb thickness lines, and the axis of the 500 mb trough was about 200 miles west of the centre. Deepening of the depression was therefore to be expected, but it was the rapidity of this deepening which poses the problem.

In the 12 hours which followed the situation illustrated in Fig. 1, the central pressure of the depression fell by 15 mb to a little below 980 mb. By 0300 GMT 29 July when the centre was just off the north coast of Cornwall, surface winds of 40 kt were being reported from all sides of the centre except the northeast. The gale reached 58 kt at the Lizard and a gust of 87 kt was recorded. Considerable damage was caused. Additional details are given by D. M. Houghton (1956).

Calculations of the expected changes in 1000 mb and 500 mb heights were carried out by numerical forecasting methods over 24-hour intervals starting from the synoptic situation at 0300 GMT 28 July and from that at 1500 GMT 28 July. The two-parameter model described by Sawyer and Bushby (1953) was used, and the technique followed that described by Bushby and Hinds (1955) in which allowance is made for warming of the air by heat from the sea. Although both of these calculations successfully indicated the larger scale synoptic

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changes, the north-east movement of the depression and the eastward displacement of the 500 mb and thickness troughs, neither of them gave any indication of the phase of very rapid deepening during the evening and night of 28 July. Both of the computations indicated a deepening of about 6 mb over 24 hours and the correlation coefficients between the calculated and observed 24-hour changes of surface pressure were 0.90 and 0.81 in respect of the two computed charts.

3. The problem of prediction in small depressions

Inspection of the relevant synoptic charts shows that the rapid deepening of the depression was a phenomenon confined to an area of 200 to 300 miles radius around the centre. Outside this region the pressure changes were not unexpectedly large and the errors of the numerical predictions were not important.

Techniques of numerical weather prediction based upon geostrophic winds and a simple two-parameter model of an adiabatic atmosphere were not designed for such relatively small scale phenomena. One might expect that to describe properly developments on this scale it would be necessary to formulate a new model, abandoning the geostrophic condition, considering the release of latent heat and elaborating the simple vertical distribution of wind assumed in the two-parameter model. However, before attempting to reformulate the problem in such a general way, there are advantages to be gained by examining whether the simple two-parameter predictions can be brought into closer accord with the observed developments by relatively minor modifications of computing technique. Besides possibly providing a "short-cut" to a practical technique, such a procedure can throw light upon the physical and mathematical factors which are important.

It is the results of computations with such a modified two-parameter model which are reported in the present paper. The calculations were all carried out over the 12-hour period of maximum deepening (namely 1500 GMT 28 July to 0300 GMT 29 July).

The principal effect which has been examined is that of the release of latent heat during condensation and precipitation, particularly in regard to its effect on the static stability of the air. The air over England on the 28th was notably warm and moist and during the period of active deepening of the depression an extensive area of heavy rain covered eastern Ireland and St. George's Channel. Rainfall amounts exceeded 40 mm in western Wales and 15 mm at Roches Point in southern Ireland. Thunderstorms were also reported in the rain area. It thus seems that a warm and moist air mass was ascending in this area under conditions in which the lapse rate differed little from the saturated adiabatic. The significance of these conditions will be discussed further in the following section together with the theoretical basis of the method adopted to examine their effects.

A second important reason why the numerical calculations might fail in respect of a depression with a small and intense core, such as that occurring on 28 July, lies in the occurrence of non-geostrophic motion. The ageostrophic wind component can be expected to become larger when the scale of disturbance is made smaller. A comprehensive treatment is however beyond the scope of the present paper but some allowance for non-geostrophic effects will be made by use of the "balance equation".

A third reason why the first calculations might have proved inadequate lies in the mathematical technique for solution of the equations. This was based on finite differences on a grid of about 150 miles mesh-length. The whole phenomenon of intense deepening thus falls within the length of four grid intervals, and may be inadequately described by so coarse a grid. For this reason some experiments were carried out using a finer mesh over the critical area.

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4. The effects of ascent of moist air on numerical forecasts

It is characteristic of active fronts and other intense synoptic features of relatively small scale that the vertical motion, as evidenced by the rainfall, is much greater than that computed by the quasi-geostrophic methods of numerical forecasting; the generation of cyclonic vorticity is also greater. Sawyer (1956) considered the vertical circulation at fronts on the basis of non-geostrophic motion perpendicular to the front, but making various simplifying assumptions about motion parallel to the front. This discussion showed that the large vertical velocities could be explained if, within the cloud mass, the limiting condition for inertial instability was approached, namely $(\partial u/\partial y)_{\theta'} = -f$ where f is the Coriolis parameter and $(\partial u/\partial y)_{\theta'}$ is the (anticyclonic) wind shear measured along a surface of constant wet-bulb potential temperature (θ' = constant). The significance of this condition is that a parcel of air displaced along a surface of constant wet-bulb potential temperature is in equilibrium at its new location both as regards hydrostatic equilibrium (it has the same density as the environment) and also in respect of dynamical equilibrium, because it has the same velocity as the environmental air at its new location. There is no restoring force tending to return the displaced parcel to its original or any other location.

An essential feature of the two-parameter models for numerical forecasting is contained in the thermodynamic equation which takes into account the effect of ascending motion in cooling the atmosphere and of descending motion in heating it. If, however, upward motion takes place under the conditions postulated in the preceding paragraph no such cooling takes place. Moreover the slope of the surfaces of constant wet-bulb potential temperature is fairly steep (say 1 in 30) and, as a first approximation, we can maintain the pattern of divergence assumed in the two-parameter models in which the convergence in the lower atmosphere is balanced by divergence in the upper troposphere directly above it, ignoring the tilt of the line joining areas of convergence and divergence which should accompany the motion up the sloping wet-bulb potential temperature surfaces.

Thus in order to study the effect of ascent of saturated air in areas of neutral inertial stability (and probably in areas of inertial or hydrostatic instability) upon the large-scale dynamics of the horizontal motion, it appears possible to treat the problem as if vertical motion had no effect upon the temperature field at points lying within the region of neutral inertial stability. Mathematically expressed, this implies that at such points the equation given by Sawyer and Bushby (1951) to determine the rate of change of their thickness parameter h', namely (in their notation):-

$$\nabla^2 \frac{\partial h'}{\partial t} + J\left(h_m, \frac{\beta}{f}\nabla^2 h'\right) + J\left(h', \frac{\beta^2 g}{f}\nabla^2 h_m + f\right)$$

$$= \frac{4g}{RA\Gamma_p}\left(\frac{\beta^2}{f}J(h'h_m) - \frac{\partial h'}{\partial t}\right)\left(\nabla^2 h_m + \frac{f^2}{\beta^2 g}\right) \qquad (1)$$

should be replaced by an equation expressing simply the horizontal advection of thickness. The appropriate equation is

$$\frac{\partial h'}{\partial t} = \frac{\beta^2 g}{f} J(h', h_m) \qquad (2)$$

The vertical velocity then has to be obtained (if required) by the evaluation of $\partial p/\partial t \equiv \Pi_m$ from the equation

$$\frac{4\Pi_m \left(\nabla^2 h_m + f^2/\beta^2 g\right)}{p_0 - p_1} = \nabla^2 \frac{\partial h'}{\partial t} + J\left(h_m, \frac{\beta^2 g}{f} \nabla^2 h'\right) + J\left(h', \frac{\beta^2 g}{f} \nabla^2 h_m + f\right) \qquad (3)$$

It is, of course, necessary in any practical computation to determine the points of the grid at which equation (2) is to be used in preference to equation (1). In the study of the depression of 28 July, equation (2) was used for three fixed points of the computational grid which fell within the area of extensive rain throughout the period of 12 hours through which the calculation was continued. There seemed good reason to think that the postulated conditions occurred in that area. Equation (2) was applied to calculate $\partial h'/\partial t$ at these points and subsequently equation (1) was solved by the modified Liebmann process of iteration regarding the points for which $\partial h'/\partial t$ had already been calculated as additional (internal) boundary points.

If such a method were used in day-to-day numerical forecasting a scheme would have to be devised by which the points for the application of equation (2) would be determined either by extrapolation of known areas of near neutral inertial stability or from the vertical motion pattern itself.

5. Results

The results of calculations of the 500 mb and 1000 mb height distributions at 0300 29 July 1956 on the basis of the corresponding distribution 12 hours earlier are illustrated in Fig. 2 and 3. (For convenience the distribution of 1000 mb height has been converted to a sea-level pressure map).

Comparison of Fig. 2(a) with 2(b) and of 3(a) with 3(b) shows the shortcomings of the calculations based on the simple two-parameter model. The eastward motion of the 500 mb trough has been slightly underestimated, and in particular values of 500 mb height are about 60 m too high over southwest England where the deep surface depression had formed. The error in the surface pressure map is more striking. The calculated chart shows a flat depression extending from south-west England to the Scottish border and contains no indication of the intense centre over Cornwall. Pressure calculated over Cornwall is 995 mb and is no lower than the initial pressure in the depression off southern Ireland. On the other hand, the position of the 1000 mb isobar is essentially correct and errors outside this are of a minor character.

In order to study the possible effect of the release of latent heat in the region of near neutral stability over Ireland, the calculations were repeated following the method outlined in the previous section. Equation (2) was applied at three adjacent grid points forming a triangle covering most of the south-eastern half of Ireland. The calculated 500 mb heights were practically unaffected by the changed procedure, but an improvement was effected in the calculated distribution of sea-level pressure. The calculated pressure between southern Ireland and Cornwall was about 991 mb - about 4 mb lower than in the first calculations, but the low pressure centre was still flat and elongated towards the Irish Sea.

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In order to ascertain whether more realistic results would be obtained using a smaller computation grid, the calculations were repeated on a grid with a mesh length of about 75 nautical miles, half of that used in the previous calculations. The area covered by the calculations, roughly that illustrated in Fig. 2 and 3, was only one quarter of the area for which the previous calculations had been performed, and the changes calculated in the first calculations were used as boundary values in the computations on the smaller grid. The points at which equation (2) was substituted for equation (1) are shown in Fig. 2(c) - these form a smaller triangle than the points used in the calculations on the larger grid.

The change to the finer grid resulted in a slight further improvement in the computed results. The computed central pressure for the depression was lowered to 989 mb and a closed centre placed off south-east Ireland. At 500 mb the trough line was advanced slightly eastward but the computed 500 mb heights were nowhere changed by more than 20 m.

Full discussion of the importance of departures from geostrophic balance would be impossible without a complete reformulation of the "model" of the atmosphere which has been used as a basis for comparison. However, some allowance for non-geostrophic motion can be made by first using the "balance equation", proposed by Charney (1955), to obtain a stream function which would correspond to non-divergent flow in the initial pressure field, by secondly using this stream function in place of the geopotential in the time-integrations and finally by recomputing the final geopotential field again from the "balance equation". This technique has been used by Bushby (1956) who applied the "balance equation" at the 600 mb level only, and also by others. In the present calculations the "balance equation" was used to determine a stream function at both the 600 mb and 1000 mb levels. In a region of strongly curved and slowly changing isobars the effect of using the balance equation is somewhat similar to the use of the gradient wind in place of the geostrophic wind, and its importance in the present case is that it takes into account the sub-geostrophic wind velocities in the strongly curved contours in the region of the developing depression.

The results of the computations based upon stream functions determined from the balance equations are illustrated in Fig. 2(c) and 3(c). The smaller grid referred to in previous paragraphs was used and allowance for the release of latent heat in the rain area over southern Ireland was made in the manner previously described. The result of all of these changes in technique has been to produce computed charts which are substantially better than those produced by the simple two-parameter model (Fig. 2(b) and 3(b)). The calculated depth of the depression is 987 mb compared with 980 mb observed - a calculated deepening of 8 mb compared with the observed deepening of 15 mb in 12 hours. The depression is also computed to be almost 100 mi too far west.

At 500 mb the trough line is computed to be about 80 miles further east than with the simple two-parameter model and coincides closely with the observed position. Maximum errors are now reduced to about 40 m and occur over central England. The improvement at 500 mb arises almost entirely from the use at 600 mb of the stream function determined from the "balance equation."

As might be expected on occasions of rapid deepening, the results of such numerical computations are rather sensitive to the assumed initial distribution of pressure. Thus, although computations based on two slightly different analyses of the initial 1500 GMT both indicated no deepening of the depression when the simple two-parameter model was used, a third similar integration gave a different result when it was based on an analysis adjusted to take into account the greater depth of the centre observed at 1800 GMT - starting with a depth of 992 mb at 1800 GMT instead of 995 mb at 1500 GMT in the previous calculations. This latter integration gave a deepening of the centre of 6 mb in 9 hours.

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6. Conclusions

The experiments reported here represent only a few of those which must be made if the greatest practical help to forecasters is to be obtained from methods of numerical forecasting. One cannot adopt such modifications of the computation procedure as have been discussed here, without ensuring that they do not introduce as great errors on some occasions as they cure on others. Nevertheless, the results are interesting as suggesting that the present rather simple numerical forecasting models can perhaps be adapted to meet the needs of the forecaster whose concern is with the developments of medium-scale systems within 24 hours. They also indicate that modification of the vertical stability of the air by the release of latent heat in regions of ascent may be an important factor in accelerating the deepening of small pressure centres.

Acknowledgements

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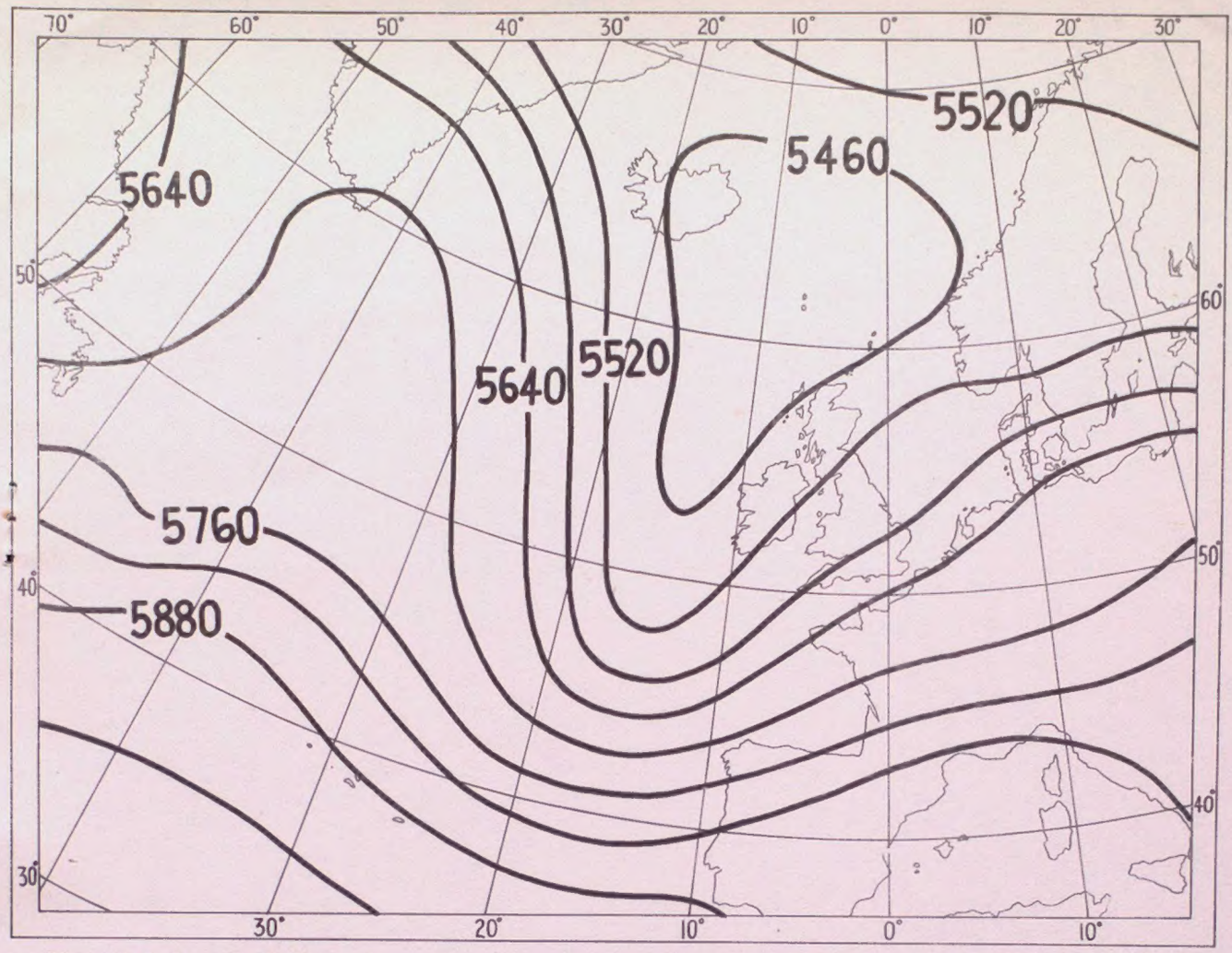

Fig.1 (a) 500 mb. contours (metres) 1500 G.M.T. 28:7:56

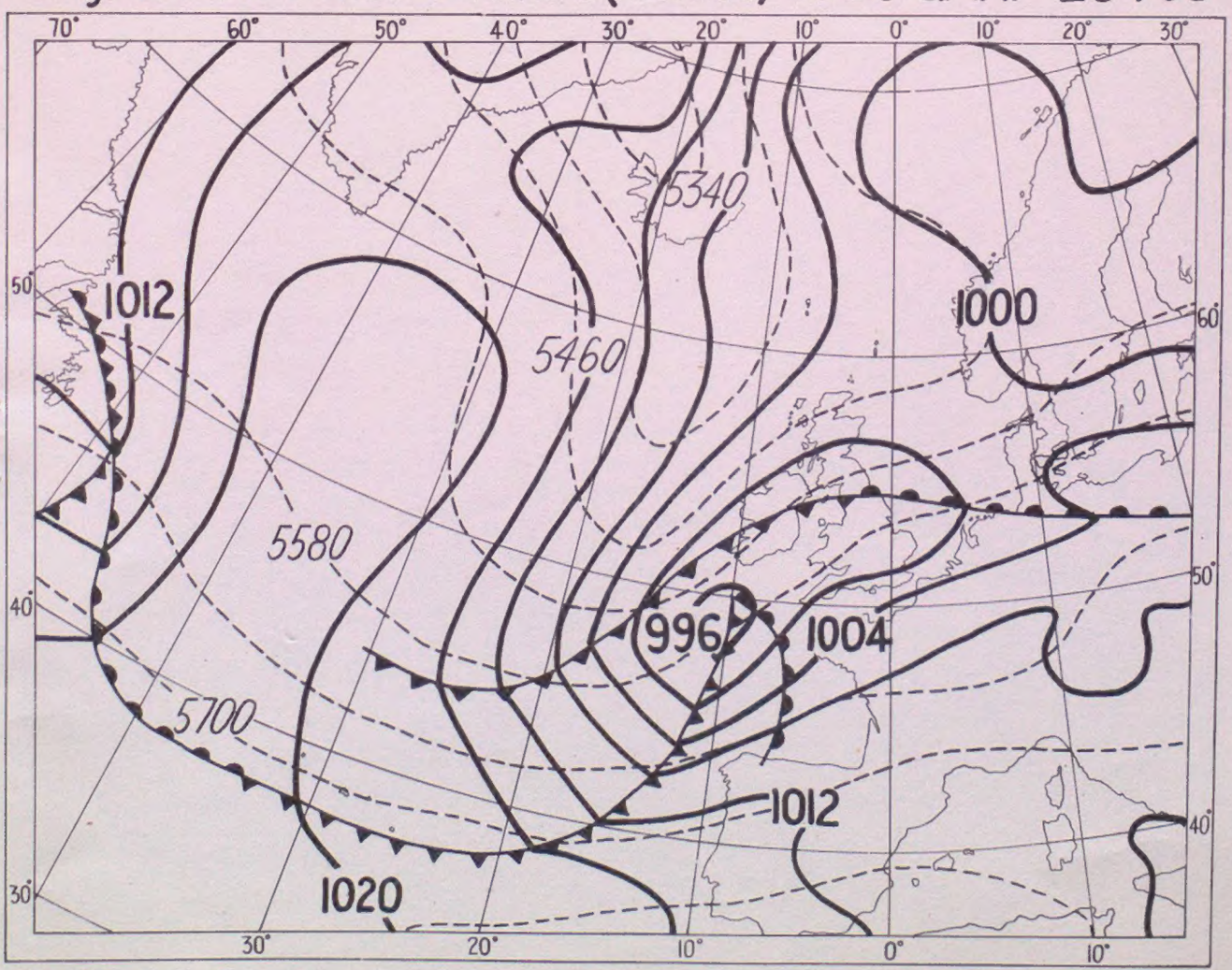

Fig.1 (b) Surface isobars and 1000-500 thickness lines (metres)
1500 G.M.T. 28:7:56

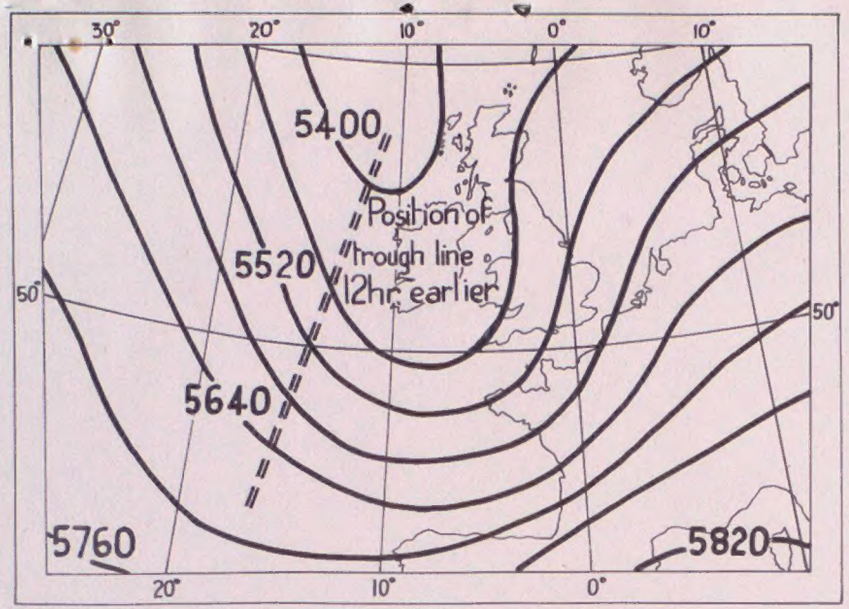

Fig 2(a) 500mb contours (metres) 0300 G.M.T. 29:7:56

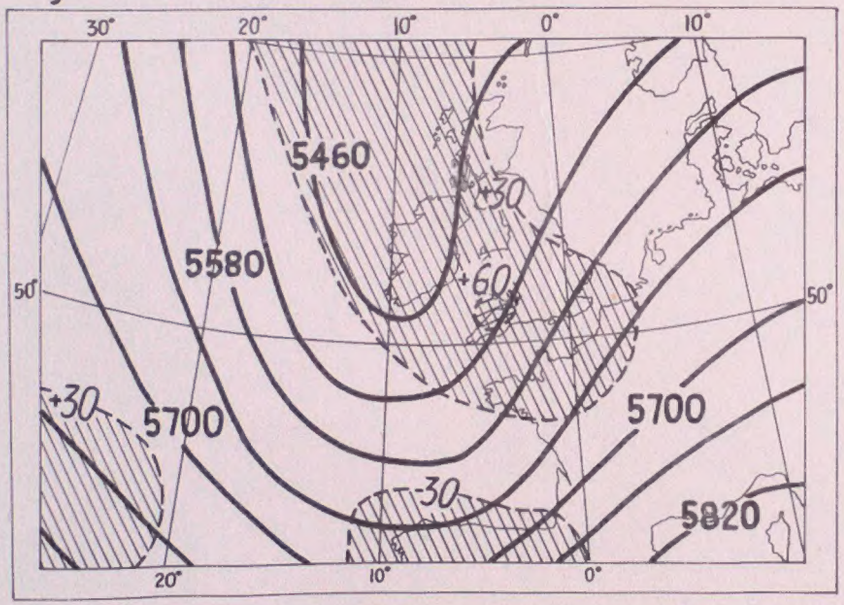

Fig 2(b) Calculated 500mb. contours (metres) 0300 G.M.T. 29:7:56 (12hr. prediction with simple two level model)

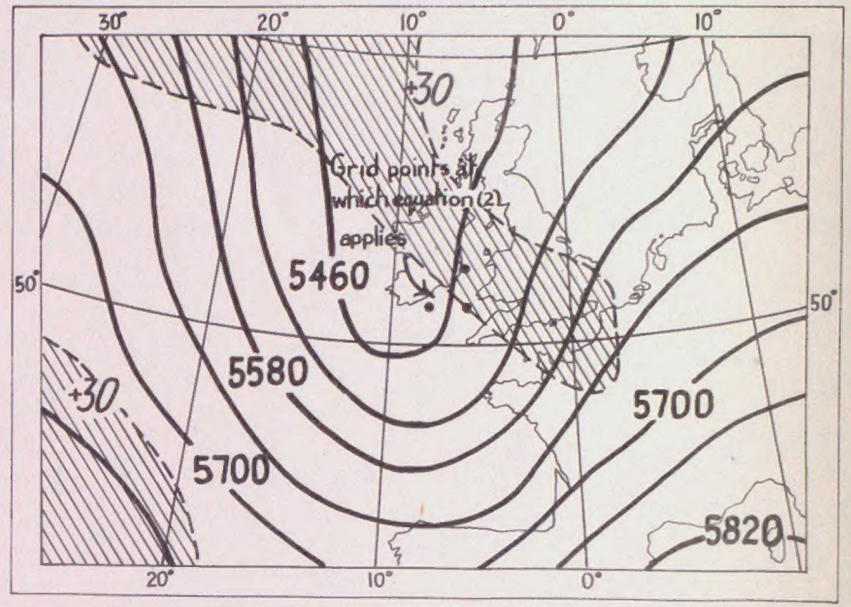

Fig 2(c) Calculated 500mb. contours (metres) 0300 G.M.T. 29:7:56 (12hr. prediction - modified stability stream function used at two levels)

Error field

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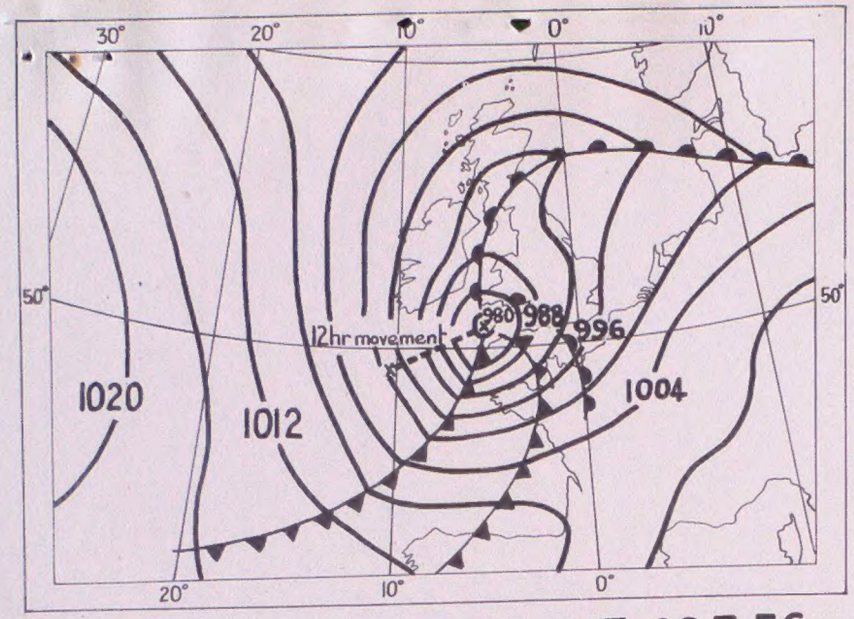

Fig.3(a) Sea level isobars 0300 G.M.T. 29:7:56

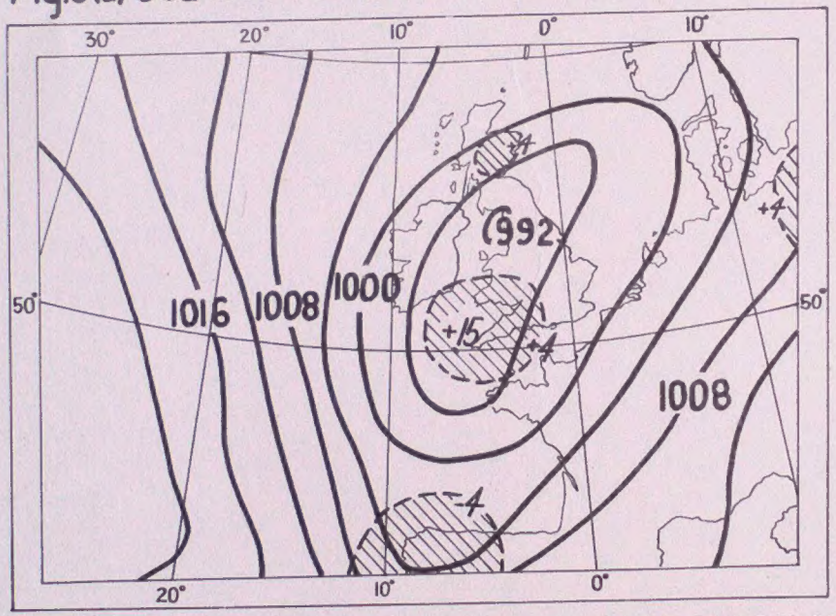

Fig.3(b) Calculated sea level isobars 0300 G.M.T. 29:7:56
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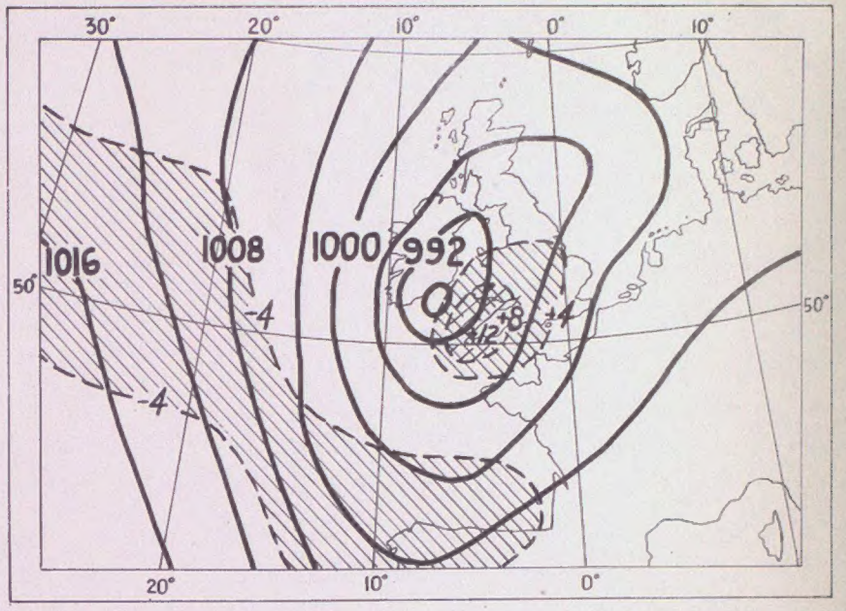

Fig.3(c) Calculated sea level isobars 0300 G.M.T. 29:7:56 (12 hr. prediction - modified stability stream function used at two levels)

Error field

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